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Structural models of complementary choices

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Abstract This article reviews the rapidly growing literature on structural models of complementary choices. It discusses recent modeling developments and identifies promising areas for future research.

Keywords Structural models · Complementary choices · Applied econometrics · Empirical industrial organization · Quantitative marketing

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1 Introduction

Complementary choices are important and pervasive yet occasionally elusive. Single consumers make complementary choices in purchase decisions (e.g., chips and salsa), product interoperabilities (smartphones and networks), and dynamic decisions (current exercise and future health-care consumption). Multiple consumers make complementary choices when they interact in strategic games or form networks. Firms make complementary choices when determining production inputs, entering related markets, and strategic mergers.

The structural empirical literature has recently started to address the difficult problem of how to model complementary choices. This new work contrasts with traditional approaches such as discrete choice models, wherein all choices are mutually exclusive.

A naïve approach to modeling complementary choices is to include all possible bundles of choices in the choice set. However, for any given set of options, the set of all possible subsets is exponentially larger and often too large to feasibly estimate. Second, specific models of complementarities are needed to ensure desirable equilibrium properties in games among agents (e.g., existence, uniqueness, or multiplicity). Third, models of complementarities are often required to evaluate counterfactuals, such as predicting demand for bundles of complementary products that have not previously been offered.

We review the literature selectively, summarizing the state of the art and identifying promising directions for future work. We begin with complementary choices made by consumers, and then examine complementary choices made by firms.

2 Demand complementarities

Complementary goods are defined using two subtly different approaches, both based on the idea of a positive interaction between the goods. Milgrom and Roberts (1990) defined complements occurring when consumer utility functions are supermodular in their arguments, i.e., for two N-dimensional vectors of complementary goods $x=(x_1,\ldots,x_N)$ and $x'=(x_1',\ldots,x_N')$, a smooth utility function satisfies:

$$u(x)-u(x')\geq\sum_i\left[u(x_i,x'_{-i})-u(x')\right]$$

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With smooth utility functions, this definition is equivalent to positive cross-partial derivative of utility with respect to quantities.[1]

The textbook definition of complements is based on negative cross-price elasticity of demand between two goods, i.e., an increase in the price of one good will result in a decrease in demand for the other (i.e., a positive cross-partial derivative of Hicksian demand). This definition may arise from a single-agent model or a theoretical characterization of aggregate demand.

Demand-side complements fall into the following non-exclusive categories:

1. Quantity complements: Higher quantity of one product leads to higher value for another, e.g., left shoes and right shoes.
2. Quality complements: Higher quality of one product leads to higher marginal value of quality for another, e.g., a suit paired with a tie.
3. Within-category complements: The value of a portfolio of related products is the option value of consuming the best fit to current needs, e.g., a home movie library.
4. Cross-category complements: When products are materially combined to obtain higher consumer value, e.g., milk and cereal, hardware and software, etc.
5. Provider-driven complementarity: Independent products become complements when provided by a single firm due to brand or service delivery spillovers, e.g., banking and investments.
6. Dynamic complements: Choices that are substitutes in static settings can become complements in a dynamic setting, e.g., different seasons of a television show.
7. Complementarities across individual agents: When agents interact, their choices may be complementary, e.g., the decision to form a relationship must be mutually chosen.

We begin with the categories that have been studied most frequently (1–5), those describing complementarity choices made by an individual decision-maker. We then consider complementary choices over time (category 6) and choices made by multiple agents (category 7).

3 Single-agent, static choices

Canonical examples of complements include jointly consumed products such as detergent and softener or chips and salsa. Estimating complementarities in aggregate demand dates back, at least, to Sato (1967), who specified a multi-level constant elasticity of substitution model and applied it to aggregate data. Causal inference is typically more difficult with aggregate data: if demand for detergent and softener are found to be positively correlated, is that because they are complementary or because demand for each product is positively related to some unobserved variable?

Most papers that have estimated complementarities using individual-level data have extended the indirect utility models underlying traditional choice models, reviewed in Manchanda et al. (1999). This approach relates the purchase incidence of a product in

[1] A related definition based on super-additivity comes from Brandenburger et al. (2011). It defines complementarity as value from product A increases with availability of product B.

one category to purchases of other products in other categories, e.g., if a consumer is more likely to buy detergent, then she may also be more likely to buy softener during that shopping trip. However, these indirect utility models typically do not specify the corresponding direct utility structure, making the assumptions about consumer preferences unclear. For example, the indirect utility function that is taken to the data might not exhibit such basic properties as homogeneity of degree zero in prices and income or quasiconvexity in prices and income.

A smaller stream of literature has derived individual-level models of complementary choices from first principles (e.g., Kim et al. 2002; Chan 2006; Gentzkow 2007; Bhat 2008; Vásquez and Hanemann 2008; Bhat and Pinjari 2013; Musalem et al. 2013). These models rely on classical economic choice theory, usually assuming that each agent maximizes a linearly additive utility function subject to a budget constraint. Applications of these models face the following primary challenges: (i) modeling both purchase incidence and quantities for multiple choices, (ii) large choice sets, (iii) determining the set of complementary goods, and (iv) balancing model flexibility and parsimony.

3.1 Multiple goods and quantities

Complementary choices may lead consumers to purchase multiple varieties of multiple goods; for example, a consumer might buy several different jars of salsa and multiple bags of chips. Traditional choice models focused on whether or not the consumer made a purchase, which Hanemann (1984) extended to incorporate quantity choice.

The direct utility approach is particularly well suited for estimating positive quantities demanded of multiple goods, because each good is associated with its own first-order condition. If utility remains quasiconcave, it is desirable to allow demand for two complementary goods to be interrelated through their purchased quantities. For example, buying more chips would increase demand for salsa. Thus, we should model not only incidence but also purchase quantity (e.g., Kim et al. 2002). For example, Lee and Allenby (2013) showed how to incorporate discrete package sizes into a direct utility model. Most consumer product categories admit only a few different package sizes; for example, in the US beer market, the most common options for beer are 40, 72, or 144 fluid ounces.

3.2 Large choice sets

A second challenge is associated with the dimensionality of the dataset. When estimating demand at the individual level, the size of the dataset scales with the number of consumers, choice occasions, and options. Therefore, datasets may easily contain tens or hundreds of millions of choices.

Moreover, choice datasets are overwhelmingly comprised of zeros (i.e., non-chosen alternatives). Therefore, our demand models must allow for corner solutions. In direct utility models, Kuhn-Tucker conditions lead to inequality constraints on utility shocks of non-chosen goods. If an agent does not choose an alternative, then marginal utility must be small, giving an upper bound inequality on the associated error. These inequalities provide a mass-contribution to the model likelihood as opposed to interior points that lead to a density contribution to the likelihood (Satomura, et al. 2011).

Moment inequalities might be a promising approach here (Pakes 2010; Figurelli 2012). Broadly, improving the computational efficiency of such models remains a promising direction for future research.

3.3 Determining the set of interrelated goods

Most of the literature identifies complementary choices a priori, with data used to measure degree of complementarity. When unexpected complementarities drive purchases, can we devise models that can easily test for the presence of hidden complementarities? A crucial challenge is that the number of complementarities grows in the number of product pairs, i.e., N goods admit N(N-1) possible pairwise complementarities. Currently, institutional knowledge typically restricts the set of possible complementarities, but tractable approaches to estimating complementarities would be helpful.

3.4 Modeling challenges

The basic identification logic requires that the likelihood of choosing B depends on whether A is chosen, or whether the individual has chosen A in the past (inventory). Berry et al. (2013) established the non-parametric identification of a continuous demand system when differentiated products are substitutes, but only for limited cases with complementarities. Establishing identification remains a technically challenging problem but is a critically important direction for future work.

Individual-level choice data is typically shallow but broad, with a small (e.g., <15) number of choice occasions per respondent. Choice attributes such as prices, package sizes, and merchandising variables change across shopping visits, requiring models that can reconcile individual-level response to these variables. Disaggregate demand models require large datasets to reliably estimate flexible models of the relationship among varieties (Allenby et al. 2005). Low-dimensional restrictions, such as allowing complementary behavior only through summary variables (e.g., category inventory), reduce the number of parameters. Furthermore, small-sample inference, coupled with discrete demand, lends itself to the use of Bayesian estimation methods (Rossi et al. 2005). Data augmentation is particularly helpful in dealing with discontinuities in demand space (Lee and Allenby 2013).

To estimate complementarities in a general form, direct utility models need to be extended, potentially leading to a curse of dimensionality, where the number of parameters to be estimated grows faster than data. This problem arises in the models of Gentzkow (2007), Song and Chintagunta (2007), Vásquez and Hanemann (2008), and Bhat and Pinjari (2013) and suggests a need for approaches that allow us to parsimoniously model these interrelations.

A "summary statistic" approach is to capture the interaction by modeling the utility of one good, e.g., a salsa brand, as a function of the total inventory of goods from a complementary category, e.g., all brands of chips (Lee et al. 2013). Thus, a consumer who has a large inventory of different brands of chips might feel compelled to buy more salsa. Complementary choices can involve more than two categories. Sriram et al. (2010) characterize three related technologies, PCs, printers, and digital cameras, modeling and identifying complementarities by aggregating all products within a category, focusing on cross-category complementarities.

Another approach is to rely on weak separability (Musalem et al. 2013). Goods are grouped into subsets. The marginal utility of each product depends on the quantity consumed of the other products within the subset. However, goods belonging to different subsets are only related through the budget constraint. For example, considering four types of goods, one subset might be {chips, salsa}, and another {detergent, softener}.

Finally, the demand for a good may depend, not only on quantity, but also product attributes of another good, e.g., purchase of detergent might depend on active ingredients, rather than brand, of the softener. Formulating a model that allows attribute complementarities is an important direction for future work.

4 Single-agent dynamic choices

Dynamics often reveal additional insights about complementary choices. Key challenges in designing dynamic models include: complementarities between sequential choices, changes from substitutes to complements, and dynamic complementarities between past/future purchases.

4.1 Choice sequences

In technology, complementary products are purchased in sequence, e.g., consumers first buy a smartphone and then apps. The first choice determines the consumer's choice set for the second, with complementarities between these creating "lock-in." Derdenger and Kumar (2013) develop a dynamic framework of complementarities between compatible products, wherein intertemporal choice dependencies are driven by consumer inventory of products and accessories (e.g., digital cameras and memory cards).

Choice sequences arise in many other contexts, e.g., TV channels offer programs exhibiting complementarities across episodes. A robust pattern in TV viewership is network loyalty, deriving from viewer preferences (together with correlation in networks' offerings), viewer switching costs, cross-promotional effects, and viewer uncertainty about program characteristics. Anand and Shachar (2011) exploited individual-level viewer and advertising exposure data to disentangle these different reasons for loyalty, with implications for programming, scheduling, and umbrella brand portfolio decisions.

Health care also involves sequential complementary decisions. Lifestyle choices affect consumers' health, complementing choices of health insurance and medical care. An individual's health status, partially observable to the researcher, connects these choices (Khwaja 2010). Moral hazard and selection influence these choices, resulting in intertemporal trade-offs. Khwaja (2013) estimates a model of repeated sequential decisions regarding health insurance, medical treatment, and health-related consumption. Health status is a serially persistent variable that can accumulate or depreciate due to individual choices, leading to endogenous longevity. The "Mickey Mantle" effect of longevity on consumption and behaviors is seen, i.e., increasing life expectancy decreases consumption of harmful products.[2]

[2] Mickey Mantle (1931–95, NY Yankees first baseman) famously remarked in his 60s, "If I knew I was going to live this long, I'd have taken better care of myself."

A related challenge is the time scale of complementarities in choices. Huang et al. (2012) examine individuals balancing long-run health goals with complementary short-run consumption decisions, which occur at different time scales. Using beverage consumption, activity, and psychological needs data, they estimate a model of high frequency consumption choices with intraday changes in short-run needs and unobserved heterogeneity across individuals. The analysis provides insights for new product introduction.

4.2 Dynamics can turn substitutes into complements

Dynamic settings often reveal hidden complementarities. For instance, Lee et al. (2013b) examined how a firm should design product quality in the "freemium" setting, studying consumer decisions using data from a file hosting service. A dynamic perspective revealed complementarity between free and paid versions, as consumers first use the free version of the product, and later upgrade to premium. Therefore, the free version increased long-run sales of the premium service, showing that "static substitutes" can become "dynamic complements."

4.3 Multi-agent choices

Complementary choices are encountered when there are network effects (e.g., Park and Town 2012). In the early literature, the payoff of a choice depended on the fraction of the population choosing the same action (see Shy 2011), e.g., likelihood of adoption depends on the user base. This modeling framework is simple, because the interaction is modeled as a one-dimensional object. More recent literature explicitly models the networks among players: the payoff of an action depends on others' actions within the local network (Kumar et al. 2014). Early empirical models recognized identification issues (Manski 1993; Bramoullé et al. 2009), including endogeneity of the connections. When the network formation is not modeled, the estimated network effects are biased due to homophily (Badev 2013; Goldsmith-Pinkham and Imbens 2013).

The formation of referral networks among physicians, or creation of social relationships in Facebook involve complementary choices: the utility of each agent's choices depends on the number and types of other agents participating. Choosing to form a link requires active choices made by both agents who share the link and benefit from both direct and indirect connections, implying strategic complementarity. Some key challenges when studying demand complementarities in networks include defining a theoretically founded modeling framework and identification of structural parameters (Jackson 2008).

Structural network formation models suffer from a curse of dimensionality: the number of possible network configurations increases exponentially with the number of players, posing severe challenges to structural estimation. The likelihood in these models involves high-dimensional integration over all possible networks (Mele 2013) or matchings (Imbens et al. 2010). Estimation interleaves parameter and network simulations to avoid the high-dimensional integration, allowing tractable estimation with myopic agents.[3] Alternative approaches develop consistent estimators based on

[3] Similar methods for dynamic discrete choice models include Ching et al. (2009), Norets (2009) and Gallant et al. (2013b).

asymptotic approximations for large networks (Chandrasekhar and Jackson 2012), or focus on partial identification and set estimation (Miyauchi 2012; DePaula et al. 2011). Pairwise stability of the network reduces the curse of dimensionality because it allows estimation using sub-networks (Sheng 2012). Leung (2013) developed a two-step estimator for a game of network formation. Badev (2013), Hsieh and Lee (2013) and Goldsmith-Pinkham and Imbens (2013) provide the first attempts to study the endogenous network formation and decisions of connected individuals.

5 Supply complementarities

5.1 Implications of demand complementarities

In contrast to notions of "core competence" that lead firms to narrower focus, demand-side complementarities encourage an expansive mindset toward firms' portfolios and boundaries, e.g., tire manufacturers were early investors in paved roads (Brandenburger et al. 2011). Demand-side complementarities have implications for pricing decisions such as "when" and "where" to make money (e.g., razor/blades or iPod/iTunes pricing), firm boundary decisions, product portfolio choices, umbrella brand strategies, and entry decisions (e.g., new versions).

Bundling, theoretically explored in static settings with non-additive valuations by Armstrong (2013), is an important strategy that requires knowledge of complementarities. In a dynamic context, bundling can enable better segmentation and price discrimination. Derdenger and Kumar (2013) found video game bundling effective in dynamic settings because it did not allow for intertemporal sorting of consumers. Further, based on intertemporal variation in the tying ratio (average software units per hardware owned), they identified the correlation of utility between two product categories with multiple sequential purchases, e.g., hardware followed by a library of software purchases.

5.2 Complementarities between firm activities

Complementarities are related to long-standing theories of "fit" in business strategy, revived by Milgrom and Roberts (1990) and Porter (1996). Consider Walmart's unique set of complementary choices: (i) rural store locations (reducing head-to-head competition with rivals), (ii) fully owned warehousing/distribution centers, (iii) information technology, and (iv) "every-day-low-pricing." Although one activity may be easy for rivals to mimic, copying the full set of complementary activities is harder, effectively creating "barriers to imitation." Complementarities can therefore explain persistent performance differences between firms ("sustainable competitive advantage").

Empirical work on supply-side complementarities falls into two streams. The first stream infers complementarities from observed adoption patterns of different activities. Cockburn et al. (2000) demonstrated complementarity between research incentives and importance of scientific publications for career advancement. Anand and Khanna (2000) found that firms learned by experience in technology licensing contracts. Novak and Stern (2009) found similar complementarities across decisions to vertically integrate different automobile systems. Gallant et al. (2013a) estimated a dynamic game of generic pharmaceutical manufacturer entry into markets for expiring patented medicines. Complementarities across markets for different products arose due to spillovers of entry on future capabilities (e.g., FDA approvals) and hence to experience in bringing generics to market. Spillovers are modeled as firm specific latent costs depending on past entry decisions. Each entry reduced costs 7 % at the next entry opportunity. Groeger (2012) finds dynamic complementarities in sequential procurement auctions, with participation in an earlier auction affecting future participation.

A second stream examines differences in performance between firms that adopt individual activities versus sets of activities, e.g., Bresnahan et al. (2002) studying complementarities in information technology and human capital investment. Related studies of human resource management include Ichniowski et al. (1997) and Ichniowski and Shaw (1999). Porter and Siggelkow (2008) noted that there is relatively little research on the degree to which complementarities depend on other choices of the firm.

5.3 Complementarity and market structure

Market power embodied in pricing incentives is a first-order consequence of mergers and acquisitions (Farrell and Shapiro 1990). On the demand side, after the Telecommunications Act of 1996, mergers increased the number of distinct programming formats in radio markets (Berry and Waldfogel 2001), while owners acquiring competing stations tended to differentiate them (Sweeting 2010). Thus, choices on firm scope or boundaries complement the product portfolio decision. Cost-side complementarities also provide a reason, with Jeziorski (2013) finding savings in operating two stations of the same format, countering the anticompetitive demand effects of mergers. Note that these cost complementarities may arise when producing products that are substitutes on the demand side, potentially leading firms to trade-off differentiation versus complementarity.

Studying the well-known question of whether dominance begets dominance (Chandler v. Schumpeter), Blevins et al. (2013) developed a dynamic oligopolistic model wherein fast food firms chose to expand or contract: size has spillovers on a firm’s future outcomes and relative dominance, industry evolution, and market structure. In fast food, they found size spillovers on market outcomes, market dominance, and structure, with heterogeneity in spillovers. McDonald’s had largest spillovers, and its ability to retain gains contributed to market dominance.

5.4 Empirical challenges

There are two major approaches for testing complementarities. The first relies upon the idea that the correlation between two complementary choices is positive, conditional on observable, exogenous factors that might impact performance. The second approach results from the idea that different levels of multiple choices (e.g., high investments in both R& D and Marketing) may interact to produce superadditive returns.

Athey and Stern (1998) provided a static framework to structure ideas about possibly complementary choices using firm-level choices about organizational decisions. They highlighted the strong assumptions that are required to draw empirical inferences from data using descriptive models. They distinguished between choices and specific sets of choices, e.g., high R& D and high Marketing investments would denote a "high–high" system. While the distinction is more apparent for discrete choices, the principle applies to continuous choices.

Ordinary least squares (OLS) and 2-stage ordinary least squares (2SLS) are unbiased only under very restrictive assumptions regarding the nature of the unobservable factors. When there is a choice-specific unobservable that affects productivity, they demonstrate that both OLS and 2SLS provide biased estimates of complementarity effects. The firm is modeled as optimizing over the set of choices, according to a performance measure. It is useful to note that this bias does not even require the presence of system-level unobservables, or even choice-specific unobservables impacting performance. Rather, the unobservables only need to have an effect on adoption of the choice.

Athey and Stern (1998) provided conditions under which the bias in the estimated complementarity can be signed. When productivity is affected by an unobservable choice-specific shock independently distributed for each choice, both OLS and 2SLS underestimate the true magnitude of complementarity and may even estimate the wrong sign. If choice-specific shocks are positively correlated across choices, they also showed that there is a fundamental identification problem whereby the inference will indicate positive complementarity where no complementarity (or even negative complementarities) might represent the true condition.

Finally, reduced form models exploiting exclusion restrictions can be used with some confidence with two choices. However, when there are three or more choices, such tests may yield estimates of complementarities between a pair of choices even when such choices are substitutes, if there is a third choice that complements one from the pair substitutes for the other one.

6 Conclusion

Complementary choices are prevalent and increasingly important. This paper identified issues that arise in structural estimation of complementary and promising areas for future research. Further progress in these directions will help to explain the determinants of demand complementarities and offer insights about how they affect business strategy and public policy. We might conceptualize the firm as a unique nexus of complementary activities, providing new insights into market structure, competition, and regulatory policy.

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